VELOCITY DISTRIBUTION OF NON-NEWTONIAN SUSPENSIONS

ABSTRACT

Velocity distributions in the laminar, transitional and turbulent flow of aqueous bentonite suspensions were experimentally investigated, whose non-Newtonian behaviour was well expressed by the model of Briant (1).

Experimental data concerning a dilute suspension \( (C = 0.032) \) satisfactorily agree with known correlations based on the Prandtl and v. Karman mixing length theory, if the viscous effects are evaluated by asymptotic plastic viscosity. Suspended particles give rise to v. Karman coefficients lower than in the case of water in low ranges of the Reynolds number.

For most concentrated suspensions the laminar flow velocity distribution was correlated by the theoretical treatment due to Le Fur & Martin (1967). Transition from laminar to turbulent flow was evidenced by velocity profiles gradually tending to the velocity distribution typical of turbulent flow of Newtonian fluids. The validity of generalized correlations for non-Newtonian turbulent flow was also verified. From experimental velocity profiles a level of turbulence lower than in water was deduced, which can imply that a thicker laminar sublayer occurs in the flow of most concentrated suspensions.

1. INTRODUCTION

Viscous properties and hydraulic resistance of aqueous bentonite suspensions in smooth and artificially roughened pipes were already investigated (Ippolito, 1972; Ippolito, 1974; Ippolito & Russo Spena, 1977).

The viscous behaviour of tested suspensions agrees with the model of Briant:

\[
\tau = \eta_\infty \dot{\gamma} \left( 1 + \frac{\tau_\infty}{\eta_\infty \dot{\gamma}} \right)^n
\]

as suggested by Le Fur & Martin (1967) for drilling muds and various suspensions. In partial ranges of the shear rate viscous properties were also checked by the power law:

\[
\tau = K \dot{\gamma}^n
\]

whose validity as rheological model for suspensions with concentration \( C \geq 0.10 \) in most range of experience was ascertained.
The experimental results concerning the friction resistance the influence of the viscous non—Newtonian behaviour in the laminar and transition regions have evidenced, whose entity was theoretically treated and experimentally verified. Fully developed turbulent flow conditions were also investigated and the validity of generalized correlations for Newtonian and non—Newtonian flows in smooth and rough pipes was ascertained.

The laminar flow velocity distributions for fluids of the Brant type in circular pipes was theoretically treated by Le Fur & Martin (1967). Turbulent flow correlations for velocity profiles in smooth pipes were suggested by Dodge & Metzner (1959), Bogue (1960), Bogue and Metzner (1963) for generally viscous non—Newtonian fluids whose rheological behaviour was correlated by the power law (2).

An experimental research program was carried out in Naples to ascertain the velocity distribution of aqueous bentonite suspensions in smooth pipes in the laminar, transitional and turbulent flow regions, both to check the correlations already proposed and to explain the influence of suspended particles on transition from laminar to turbulent flow. Full results were already published (Ippolito & Sabatino, 1985).

2. EXPERIMENTAL PROCEDURE

The velocity distribution of tested flows was investigated by a stainless steel dynamic probe of the Pitot type, whose position was guided by a driving system into the test section at the end of the experimental pipe (internal diameters 51.3 mm and 71.8 mm). The probe had an internal diameter d_i = 2 mm and external diameter d = 2.65 mm. Experimental test section was also provided with a static probe at the wall.

The test fluid was circulated by a Moyno pump driven by a direct current electric motor equipped with a fine regulation system.

The viscous properties of tested suspensions, pressure losses in the experimental pipes and discharges were also experimentally obtained.

Nearly constant temperature operating conditions were assured by means of a heat exchanger connected to a refrigerating equipment.

The coefficient of the dynamic probe:

\[ c_p = \frac{P_0 - P}{\frac{1}{2} \rho v^2} \]  \hspace{1cm} (3)

was evaluated by:

\[ c_p = 1 + \frac{28.3}{Re_p^{1/2}} \]  \hspace{1cm} (4)

where, as suggested by Bogue (1960), Re_p is the Reynolds number of the probe taking account of the non—Newtonian viscous behaviour by means of the power law (2).
The accuracy of experimental velocity profiles, of the order of some per cent, was controlled by the comparison of discharges obtained both by integration of experimental velocity distribution and by Venturi meter.

3. EXPERIMENTAL RESULTS

Experimental results concern aqueous suspensions of bentonite Supergel produced by SAMIP, at the Ponza Island, in Italy. Concentrations by weight of dry solids \( C = 0.032; 0.075; 0.097 \) were investigated, the first at 29°C and the others at 19°C, in thermal equilibrium with ambient temperature.

The rheological behaviour of tested suspensions obtained by viscometric stainless steel pipes (int. dia. 3mm; 4mm) satisfactorily agrees with the model of Briant (1), as evidenced by fig. 1, where shear stresses at the wall \( \tau_o \) versus shear rate \( \dot{\gamma}_o \) are shown. Experimental \( \tau_o \) were deduced through the experimental pressure losses concerning the viscometric pipe corrected for the end effects, as explained in a previous note (Ippolito, 1972). Shear rates at the wall \( \dot{\gamma}_o \) were obtained by means of the correlation:

\[
\dot{\gamma}_o = \frac{3 + 1/n'}{4} \frac{32Q}{\pi D^3} \tag{5}
\]

where

\[
n' = \frac{d \left( \log \tau_o \right)}{d \left( \log \frac{32Q}{3\pi D^3} \right)} \tag{6}
\]

from experimental flow curves showing \( \tau_o \) versus \( \frac{32Q}{3\pi D^3} \) were deduced.

Experimental results concerning the friction resistance both for experimental Perspex test sections \( (D = 51.3 \text{ mm and } 71.8 \text{ mm}) \) and for viscometric pipes, according to a dimensionless correlation of the type:

\[
\lambda = f \left( Re_{\eta_m}; He_{\eta_m}; m \right), \tag{7}
\]

in fig. 2 are shown, where \( \lambda \) is the friction number, \( Re_{\eta_m} \) is the Reynolds number and \( He_{\eta_m} \) is the Hedström number typical of the Bingham plastic asymptotic behaviour. Experimental data concerning laminar flow conditions satisfactorily agree with the theory of Le Fur & Martin (1967), while for turbulent flow experimental data on the whole follow well enough the dashed line typical of Newtonian fluid with viscosity \( \eta_m \). Experimental range of transition agrees with the critical Reynolds number \( (Re_{\eta_m}) \), evaluated by means of the theory of Le Fur &
Fig. 1 - Rheological behaviour of tested suspensions.

Martin (1967), extending to fluids of the Briant type the criterion for transition previously suggested by Ryan & Johnson (1959).

Experimental velocity distributions typical of turbulent flow concerning \( C = 0.032 \), plotted in dimensionless chart of \( v/V^* \) versus \( y/r_o \) (fig. 3), are well fitted by the velocity defect laws:

\[
\frac{v}{V^*} = \frac{v_{\max}}{V^*} - \frac{2.3}{k} \log \frac{1 + \left( \frac{r_o}{r_i} \right)^{3/2}}{1 - \left( \frac{r_i}{r_o} \right)^{3/2}} \tag{8}
\]

proposed by Zagustin A. and Zagustin K. (1969), and:

\[
\frac{v}{V^*} = \frac{v_{\max}}{V^*} + \frac{2.3}{k} \log \left( \frac{y}{r_o} \right) - \frac{1}{k} \phi \left( \frac{y}{r_o} \right) \tag{9}
\]

proposed by Marchi (1961), where:
Fig. 2 - Experimental friction number $\lambda$ versus $Re_{re}$.

Fig. 3 - Experimental $\frac{V}{V^*}$ vs. $\left(\frac{y}{h}\right)$; $C = 0.032$. 
\[ \phi \left( \frac{r}{\rho_0} \right) = \int_{1}^{v/\rho_0} \frac{\exp \left( \sqrt{1 - \frac{Y}{Y_0}} \right) - 1}{Y/\rho_0} \, d \left( \frac{Y}{Y_0} \right) \] (10)

The values of \( v \) Karman coefficient \( 0.31 \leq k \leq 0.35 \) are lower than \( k = 0.36 \) fitting the velocity profiles preliminarily obtained for water flows. No significant influence of the non—Newtonian behaviour of suspensions on velocity distribution in turbulent flow region is evidenced.

Experimental velocity distributions for suspensions with \( C = 0.075 \) and 0.097 in fig. 4 are shown. For \( C = 0.075 \) one profile is certainly of the laminar type because its Reynolds number \( Re_{\eta_m} = 4080 \) is clearly lower than the critical Reynolds number \( (Re_{\eta_m})_c = 8320 \) previously obtained; the other experimental velocity profiles fall in the transitional or in the turbulent flow regions. For \( C = 0.097 \) three velocity profiles belong to the laminar flow region their Reynolds numbers \( Re_{\eta_m} \) being lower than the critical value \( (Re_{\eta_m})_c = 9210 \); two profiles fall in the transitional zone; fully turbulent flow conditions are not reached.

Experimental velocity distributions typical of laminar flow generally agree

Fig. 4 - Experimental velocity profiles for aqueous bentonite suspensions:

- a) \( C = 0.075 \);
- b) \( C = 0.097 \).

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with the theory of Le Fur & Martin (1967), although some deviations can be observed, perhaps due to various uncertainties concerning the evaluation of the rheological parameters and their reliability for correlating experimental flows in large pipes. The onset of turbulence by experimental velocity distributions very different from theoretical laminar ones is evidenced as shown for C = 0.075 with \(\text{Re}_{\eta_m} = 8390\).

No theoretical treatment of the velocity distribution for fluids of the Brian type in the transitional or in the turbulent flow regions is available. Therefore, the influence of the non—Newtonian behaviour on transitional or turbulent flow velocity distribution was examined by means of the correlations suggested by Dodge & Metzner (1959), Bogue (1960) and Bogue & Metzner (1963) for fluids whose viscous behaviour was fitted by the power law (2), whose parameters K and n were evaluated in partial ranges of the flow curve showing \(\tau_\lambda\) versus \(\dot{\gamma}_o\) (fig. 1).

The comparison of experimental velocity profiles with the correlation suggested by Dodge & Metzner (1959) was not satisfactory and therefore is not reported. Satisfactory agreement with the correlation suggested by Bogue (1960) & Bogue and Metzner (1963):

\[
\frac{V}{V^*} = F\left(\frac{V}{V^*}, \lambda\right) = 5.57 \log \left(\frac{\rho V^{6.5} \eta^6}{K}\right)^{1/n} + I(n; \text{Re}^*)
\]  

(11)

was obtained for velocity profiles concerning fully developed turbulent flow (fig. 5) for C = 0.075 with \(\text{Re}^* = 6220\) and 8100.

Into the eq. (11):

\[
F\left(\frac{V}{V^*}; \lambda\right) = 0.05 \sqrt{\frac{8}{\lambda}} \exp \left[-\left(\frac{V}{V^*} - 0.8\right)^{3/0.15}\right]
\]  

(12)

is a correcting function already introduced by the authors for Newtonian fluids, and

\[
I(n; \text{Re}^*) = -\frac{5.57}{n} \log \left[\left(\text{Re}^*\lambda^{1/2}\right)^{\frac{3 + 1/n}{4}} \frac{n}{64} \eta^{2.5n}\right] + 0.984\left[\frac{n}{\lambda}\right]^{1/2} + 3.63
\]  

(13)

as deduced from Bogue's thesis (1960), takes account of the influence of the non—Newtonian behaviour on the correlation between \(v/V^*\) and the specific local Reynolds number \(q V^{\infty} \eta / K\). However, the experimental velocity profile concerning C = 0.075 with \(\text{Re}^* = 8100\) (or \(\text{Re}_{\eta_m} = 19900\)) satisfactorily agrees with the Newtonian velocity defect laws (8-9) when \(k = 0.40\) is considered (fig. 6), showing the tendency to Newtonian velocity defect laws at increasing Reynolds number in fully developed turbulent flow conditions.

Referred theories for turbulent flow are based on the assumption that the turbulent shear stresses in the turbulent core and the thickness of the laminar sublayer are not substantially influenced by the links typical of suspensions, respon-
Fig. 5 - Comparison of experimental velocity profiles with the correlation proposed by Bogue (1960) and Bogue and Metzner (1963); $C = 0.075$.

Fig. 6 - Experimental $\frac{V}{V^*}$ vs. $\left(\frac{y}{\tau_0}\right)$; $C = 0.075$; $Re = 211000$; $Re_{\infty} = 19900$; $k = 0.40$.

Possible for their non-Newtonian behaviour. To verify this assumption, as it was possible, the apparent shear stresses and kinematic turbulent viscosity distributions from experimental turbulent velocity profiles were deduced.

Results concerning apparent stresses in fig. 7 are summarized, where the experimental values of $\tau_{app}/\nu V^* v_2$ versus $y/\tau_0$ are plotted. Experimental data concerning $C = 0.032$ are very near to the total shear stress linear distribution:
\[
\frac{\tau}{\varrho V^2} = 1 - \frac{Y}{r_o}
\]  
(14)

showing very small viscous shear stresses. For \( C = 0.075 \) the strong influence of the viscous effect in wide region of the pipe is evidenced; experimental apparent shear stresses distributions obtained for different velocity profiles clearly show the influence of the Reynolds number; very little turbulent stresses in most region of the pipe for \( \text{Re}^* = 2290 \) are shown.

![Graph showing apparent stresses vs. dimensionless wall distance](image)

Fig. 7 - Experimental apparent stresses \( \tau_{\text{app}} \) vs. \( \left( \frac{Y}{r_o} \right) \); \( C = 0.032 \); \( C = 0.075 \).

Kinematic turbulent viscosities concerning \( C = 0.075 \) compared with distributions obtained for turbulent flows of water, in fig. 8 are plotted. A very different form of most experimental distributions for suspensions near the wall is evidenced, which can be explained with the assumption that a thicker laminar sublayer and a transition region more detached from the wall of the pipe occurs in the flow of most concentrated suspensions.
Fig. 8 - Experimental kinematic turbulent viscosity \( \frac{\nu}{\nu^* r_0} \) vs. \( \left( \frac{y}{r_0} \right) \); \( C = 0.032; C = 0.075 \).

--- water.

4. CONCLUSIONS

As concluding remarks, from experimental results can be deduced that for dilute suspensions the velocity distribution can be defined by correlations based on the Prandtl and v. Karman mixing length theory, while for concentrated suspensions was impossible to correlate the velocity distributions with referred theories in a wide range of the Reynolds number.

Apparent shear stresses and kinematic turbulent viscosity distributions show a lower level of turbulence than in water in most range of experience concerning most concentrated suspensions, evidencing a thicker laminar sublayer in the range of transition from laminar to turbulent flow.

LIST OF SYMBOLS

C 
weight concentration of dry solids

c_p 
coefficient of the Pitot tube

D 
internal diameter of the pipe

d; d_i 
external and internal diameters of the Pitot tube
\[ He = \frac{\rho D^2 \tau_w}{\eta_w} \]  
Hedström number

\[ K \]
parameter of consistency

\[ k \]
v. Karman coefficient

\[ m \]
parameter of plasticity

\[ n \]
index of non-Newtonian behaviour

\[ p \]
static pressure

\[ p_0 \]
stagnation pressure

\[ Q \]
discharge

\[ Re = \frac{\rho V D}{\mu}; Re_{\eta_w} = \frac{\rho V D}{\eta_w}. \]  
Reynolds number

\[ Re^* = 8 \left( \frac{n}{6n+2} \right)^n \frac{\rho V^2 n D^3}{K} \]
genralized (or modified) Reynolds number

\[ Re^*_{\rho} = \frac{\rho V^2 n d^3}{K} \]
generalized Reynolds number of the Pitot tube

\[ r \]
distance from the centerline

\[ r_o \]
tube radius

\[ V \]
average velocity

\[ V^* = \sqrt{\frac{r_o}{e}} \]
friction velocity

\[ v \]
velocity

\[ v_{max} \]
velocity at the centerline

\[ y = r_o - r \]
distance from the wall

\[ \dot{\gamma} \]
shear rate

\[ \dot{\gamma}_o \]
shear rate at the wall

\[ \dot{e} = \frac{\tau_{pp}}{e \frac{dv}{dy}} \]  
turbulent kinematic viscosity
\[ \mu \] viscosity of suspending liquid

\[ \eta_\infty \] asymptotic Bingham plastic viscosity

\[ \lambda = \frac{\tau_0}{\rho V^2} \] friction number

\[ \rho \] density

\[ \tau \] shear stress

\[ \tau_{\text{app}} \] apparent shear stress

\[ \tau_0 \] shear stress at the wall

\[ \tau_\infty \] yield stress of the asymptotic Bingham plastic behaviour

REFERENCES


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